

# COOPERATIVE MATHEMATICS

*for level seven*

Robin McIntyre



| <b>TOPIC</b>                | <b>MIX AND MATCH</b>  | <b>INFORMATION SHARING</b>   | <b>SEQUENCING</b>  | <b>LANGUAGE MATHEMATICS</b>                |
|-----------------------------|---|--|--|--|
| <b>TRIGONOMETRY</b>         | Trigonometric curves p 7  | Trigonometry rules q 6,7   | Trigonometry rules p 47,48   | Circular measure p 50<br>Trigonometry p 51 |
| <b>CALCULUS</b>             | Gradients and graphs p 9  | Maximising q 8, 9<br>Equation of the tangent q 10<br>Integration:<br>Area under the curve q 11 | Optimisation p 45,46   | Calculus p 52                              |
| <b>GRAPHS</b>               | Hyperbolas p 3<br>Functions p 1<br>Parabolas/<br>Hyperbolas p 5<br>Coordinate geometry (1) p 11<br>Coordinate geometry (2) p 13 | Graph drawing q 1-5<br>Hyperbolas q 12,13  | Locating turning points of cubic functions p 36<br>Coordinate geometry p 39 ,40            | Functions p 54<br>Coordinate geometry p 53 |
| <b>SEQUENCES AND SERIES</b> | Sequences and series p 15<br>Sequences p 17   | Sequences and series q 14,15   |  | Sequences and series p 55                  |
| <b>ALGEBRA</b>              | Terminology (1) p 19<br>Terminology (2) p 21  |  | Completing the square p 35<br>Rearranging equations p 37<br>Simultaneous equations p 41,42 | Algebra p 56                               |
| <b>STATISTICS</b>           |   | Probability trees q 16-18<br>Normal distribution q 19,20                                       | Standard deviation proof p 37<br>Standard normal distribution p 43,44                      | Statistics p 57                            |

p = page number q = question number

# **Cooperative Mathematics**

**for level seven**

**Robin Averill McIntyre**



Longman Paul



## DEDICATION

I dedicate this book to my husband Trevor and my son Christopher, both of whom supported its production; Trevor, by giving much positive encouragement, and Christopher, by doing much sleeping in his first few months of life.



## ACKNOWLEDGEMENT

I would like to thank the many teachers and students who trialled these activities, particularly Margaret Priest and students of Queen Margarets College for their thorough and valuable comments. Thanks also to teachers and students of Samuel Marsden College, Upper Hutt College, Newlands College and Hutt Valley Memorial College. My gratitude goes to Tingay Davidson who introduced me to these types of activities, and to Trevor McIntyre and Ken Harrop for their very useful suggestions.

Robin McIntyre  
September 1994

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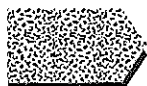
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# CONTENTS

|  |                                 |
|--|---------------------------------|
| Introduction .....                               | iv                              |
| Hints for teachers .....                         | v                               |
| <b>Mix and Match Activities:</b>                 |                                 |
| Functions .....                                  | 1                               |
| Hyperbolas .....                                 | 3                               |
| Parabolas/Hyperbolas .....                       | 5                               |
| Trigonometric curves .....                       | 7                               |
| Gradients and graphs .....                       | 9                               |
| Coordinate geometry (1) .....                    | 11                              |
| Coordinate geometry (2) .....                    | 13                              |
| Sequences and series .....                       | 15                              |
| Sequences .....                                  | 17                              |
| Algebra: Terminology (1) .....                   | 19                              |
| Algebra: Terminology (2) .....                   | 21                              |
| <b>Information Sharing Activities: .....</b>     |                                 |
| Q.1-5 .....                                      | Graph drawing                   |
| Q.6,7 .....                                      | Trigonometry                    |
| Q.8,9 .....                                      | Maximising                      |
| Q.10 .....                                       | Equation of the tangent         |
| Q.11 .....                                       | Integration: Area under a curve |
| Q.12,13 .....                                    | Hyperbolas                      |
| Q.14,15 .....                                    | Sequences and series            |
| Q.16-18 .....                                    | Probability trees               |
| Q.19,20 .....                                    | Normal Distribution             |
| <b>Single Sequencing Activities:</b>             |                                 |
| Completing the square .....                      | 35                              |
| Locating turning points .....                    | 36                              |
| Standard deviation proof .....                   | 37                              |
| Rearranging equations .....                      | 37                              |
| Coordinate geometry .....                        | 39                              |
| <b>Double Sequencing Activities:</b>             |                                 |
| Coordinate geometry .....                        | 40                              |
| Simultaneous equations (line/curve) .....        | 41                              |
| Standard normal distribution .....               | 43                              |
| Optimisation .....                               | 45                              |
| Trigonometry rules .....                         | 47                              |
| <b>Language Mathematics Activities:</b>          |                                 |
| How to use language mathematics activities ..... | 49                              |
| Circular measure .....                           | 50                              |
| Trigonometry .....                               | 51                              |
| Calculus .....                                   | 52                              |
| Coordinate geometry .....                        | 53                              |
| Functions .....                                  | 54                              |
| Sequences and series .....                       | 55                              |
| Algebra .....                                    | 56                              |
| Statistics .....                                 | 57                              |
| Answers .....                                    | 58                              |



## INTRODUCTION

This book contains activities that are designed to reinforce the mathematical concepts and skills taught in senior mathematics programmes. The activities can be used by groups of students who enhance their learning by discussing and practising the mathematical terminology and skills, while working together in a cooperative and enjoyable way.

There are four styles of activity in this book, each of which can be used in many different ways (see below for some ideas). An explanation on how to use the Language Math-





ematics Activities is included at the beginning of that section, on pages 49 and 50.

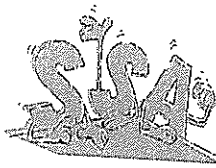
The activities are grouped according to style (see the contents page). A symbol is included at the top of each page for quick reference (see below). The table on the inside of the front cover shows the activities according to topic and style.

It is hoped that students will enjoy doing these activities, and also learn from working together with others.







### Ideas for using Mix and Match Activities

-  Match each of the graphs, equations and phrases on the left hand page with four of the numbered statements from the right hand page.
-  Write four mathematical statements for each of the graphs etc. on the left hand page, without looking at the right hand page.
-  Make up two more problems with four descriptor statements for each. Mix up the descriptors and swap with another group.
-  Use the problems in conjunction with the answers to help with revision.






### Ideas for using Single Sequencing Activities

-  Put the steps into the correct order to solve the problem.
-  Put the steps into the correct order and either write a sentence to describe the mathematics that has been used at each step or discuss what happened at each step of the problem.
-  Make two similar problems, mix up the steps, and swap with another group.
-  Use the problems in conjunction with the answers as an aid to revision.



### Ideas for using Information Sharing Activities




These problems are split into six sets of clues. Each set is on a different page and can be identified by a large letter (A-F) at the top of the page. Each group member can have one or more sets of clues. Each person shares their clues with the other group members, in order to compile enough information to solve the problems.

-  Exchange information by talking only, so that each group member can draw the graph or solve the problem.
-  Exchange information by talking only and solve the problem together on one piece of paper.
-  Make similar problems to swap with other groups.



### Ideas for using Double Sequencing Activities

Put the written descriptions (A-G) into the correct order. Match each with the symbolic part of the solution. These boxes are numbered. For each set of written instructions, except the coordinate geometry problem on page 40, there are three different problems to sequence. The coordinate geometry problem has only one problem to sequence.

-  Find one correct sequence from the answers. Use this to sequence the other part(s) of the activity.
-  Use both parts of the answer to learn how to solve problems of that type. Complete some other questions from a text book in the same way.
-  Use the sequence of written instructions to solve other problems.

## HINTS FOR TEACHERS

Use one book for each student.







These activities can be set as tasks to be done individually by students. However they were written with groupwork in mind. Groupwork encourages student discussion of the concepts involved. Whether done individually or in groups these activities provide an alternative and enjoyable means of learning level seven mathematics.

The grid on the inside of the front cover shows where to find activities in the book on specific topic areas. You can use different styles of activity on the same topic in a lesson (or series of lessons) or you can use the activities as a different kind of revision lesson.

### Ideas on how to set up groups

Three students makes an ideal group size.

There are many ways of splitting your class into groups. Here are a few:

-  straight from the roll in alphabetical order
-  people with birthdays in the same month
-  students with the same number of letters in their first name
-  specific groups chosen by the teacher using some criteria (e.g. ability, gender balanced, ethnic mix etc.)
-  draw names from a hat
-  students draw cards out of a hat. The cards each have a mathematical symbol (e.g. +, -, =, \$, % ...). There should be three of each kind of symbol. All the students with matching symbols get together.




Desks may need a quick rearrangement in order to have an ideal classroom set up. Train the class to lift the desks quietly into a group format. Leave room between the groups for the teacher to circulate and have at least three desks for each group so that they have enough space to work effectively. A quiet environment is best for group discussion and student thinking, as well as teacher sanity!!

If you wish you can have each group assign specific jobs to group members, such as recorder, chairperson, enthusiast, etc. These jobs should rotate with each activity so each group member has experience in each position.

All the activities have been trialled in 1993 by schools in the Wellington area.

## To students

If you:

-  are studying level seven mathematics
-  enjoy working with other students and/or like working on your own
-  want types of mathematical activity that are different from the conventional text book style questions

this book is for you!

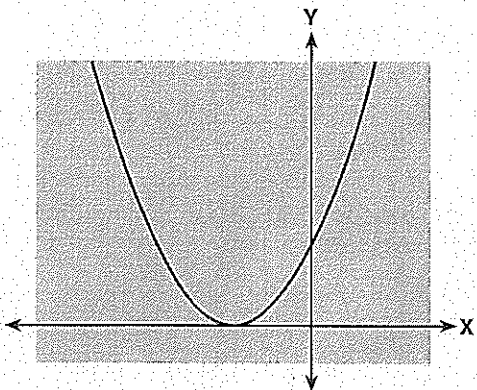
You can use this book on your own, or in groups of two, three or more. You will gain most from the activities if you discuss them with others. You can use it at home or at school. The activities are set up to help develop your understanding and enjoyment of level seven mathematics.



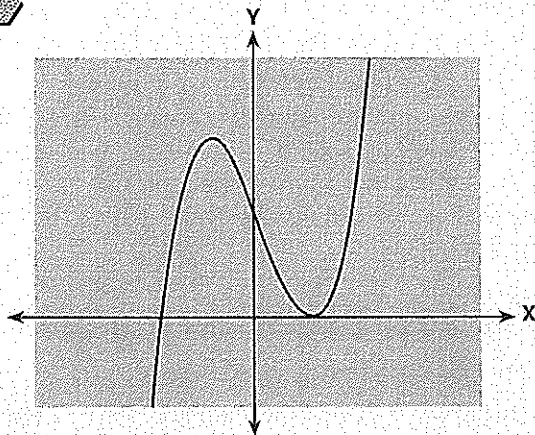
# Functions

Match four appropriate statements from page two with each graph.

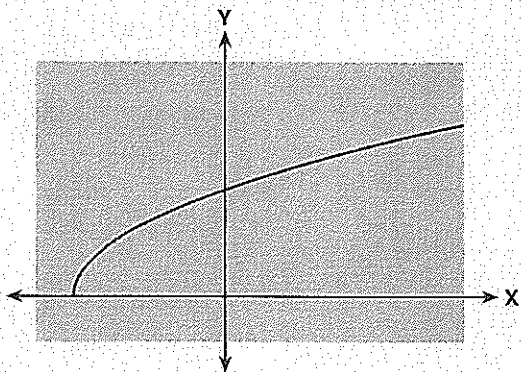
**A**



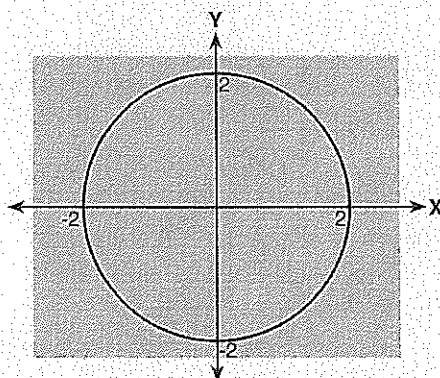
**B**



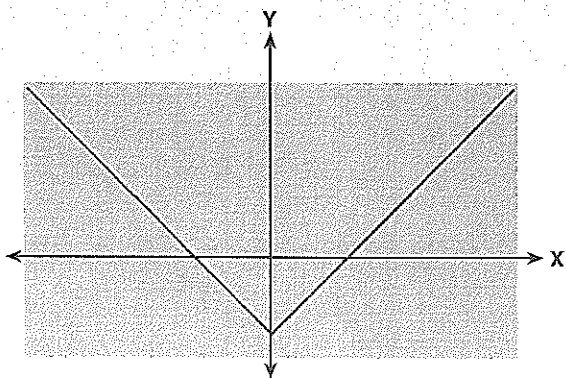
**C**



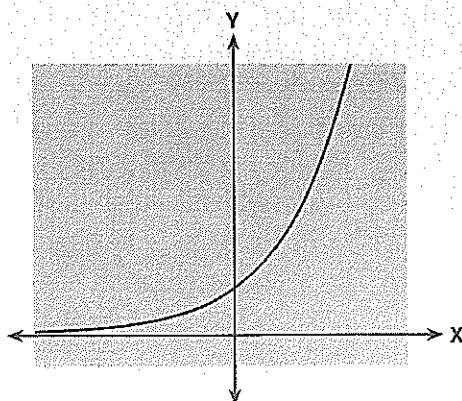
**D**



**E**



**F**





1

The minimum point is  $(-2, 0)$

2

The  $y$ -intercept is  $(0, 12)$

3

This is an absolute value function

4

This is an exponential function

5

The formula is  $y = (x - 2)^2(x + 3)$

6

The equation is  $y = (x + 1)^2$

7

The equation is  $y = 2^x$

8

The equation is  $y = \sqrt{4 - x^2}$

9

The  $y$ -intercept is  $(0, 1)$

10

This is a parabolic function

11

The equation of the curve is  $y = \sqrt{x + 2}$

12

This is a cubic function

13

The function increases when  $x > -2$

14

The function is  $y = |x| - 2$

15

This curve is not a function

16

The  $y$ -intercept is  $(0, 1)$

17

There is a local minimum at  $(2, 0)$

18

There are two  $x$ -intercepts

19

The  $y$ -intercept is  $(0, \sqrt{2})$

20

The function is decreasing when  $x < 0$

21

As  $x \rightarrow -\infty, y \rightarrow 0$

22

This is a square root function

23

The function is symmetrical about the  $y$ -axis

24

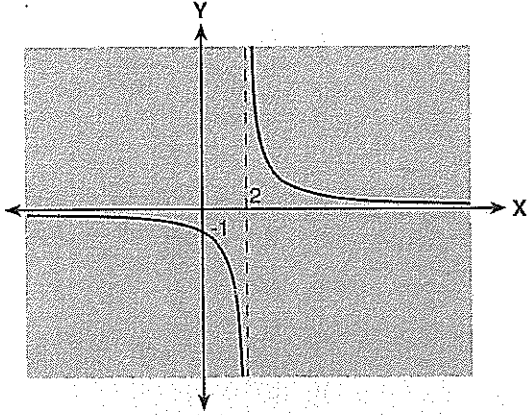
The equation is  $x^2 + y^2 = 4$



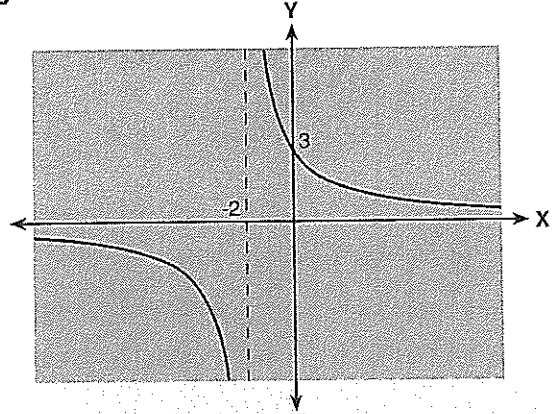
# Hyperbolas

Match four appropriate statements from page four with each graph.

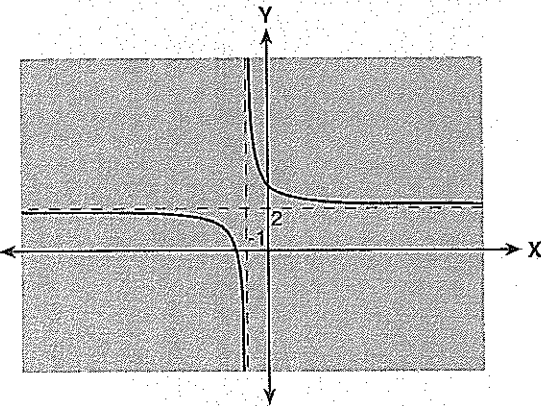
**A**



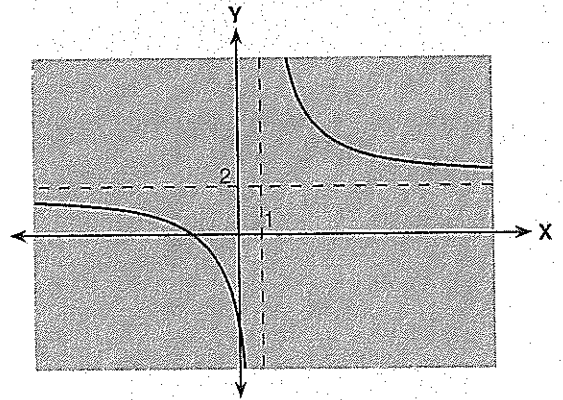
**B**



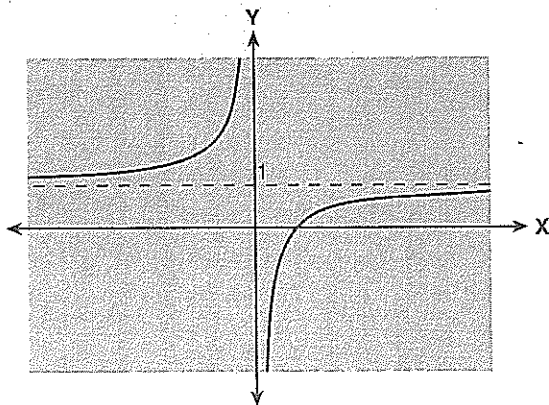
**C**



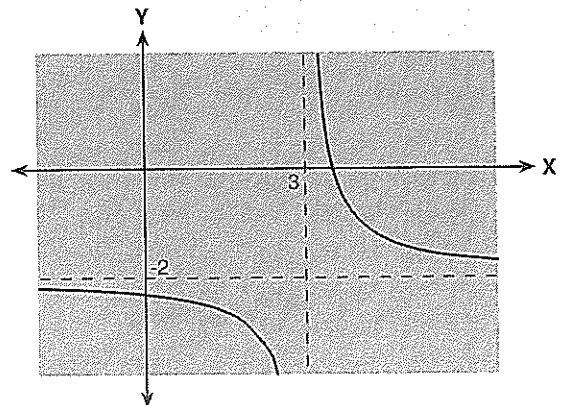
**D**



**E**



**F**



1

The y-intercept is  $(0, -1)$

2

The function is  $y = \frac{2}{x-2}$

3

The curve cuts the y-axis at  $(0, 3)$

4

The x-intercept is  $(1, 0)$

5

The graph is  $y = \frac{x-1}{x}$

6

This is a hyperbolic function

7

The horizontal asymptote is  $y = 0$

8

As  $x \rightarrow 3, y \rightarrow -\infty$

9

The equation of the curve is  $y = \frac{7-2x}{x-3}$

10

The curve is  $y = \frac{2x+4}{x-1}$

11

The domain of the function is  $x \neq -1, x \in R$

12

The horizontal asymptote is  $y = 1$

13

The vertical asymptote is  $x = 3$

14

$y = x + 2$  is an axis of symmetry of the curve

15

The x-intercept is  $(-\frac{3}{2}, 0)$

16

The function is  $y = \frac{1}{x-3} - 2$

17

The vertical asymptote is  $x = 1$

18

The function is  $y = \frac{2x+3}{x+1}$

19

The range is  $y \in R, y \neq 0$

20

This is the graph of  $y = \frac{6}{x+2}$

21

The function cuts the x-axis at  $(-2, 0)$

22

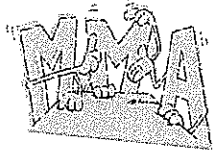
As  $x \rightarrow -\infty, y \rightarrow 1$

23

As  $x \rightarrow \infty, y \rightarrow 0$

24

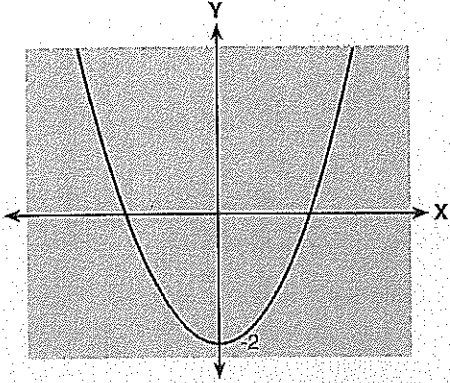
The graph is  $y = 2 + \frac{6}{x-1}$



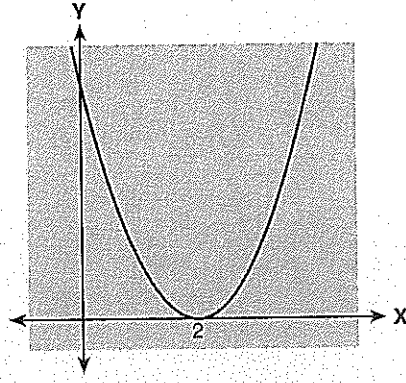
# Parabolas/ Hyperbolas

Match four appropriate statements from page six with each graph.

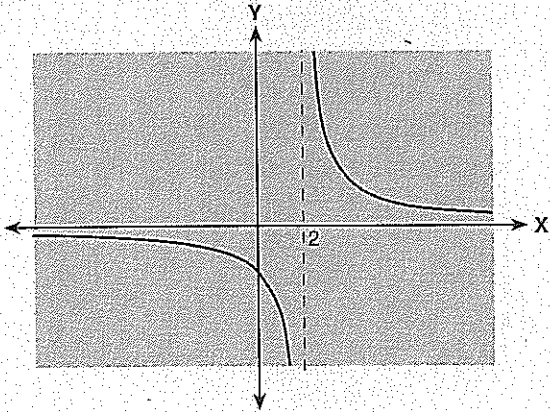
**A**



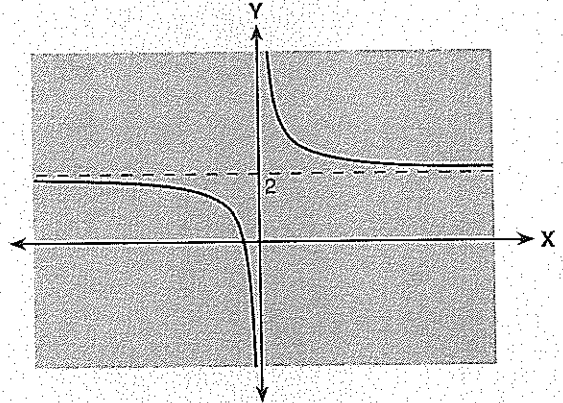
**B**



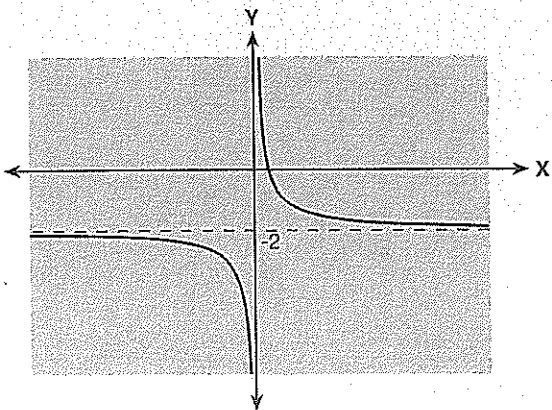
**C**



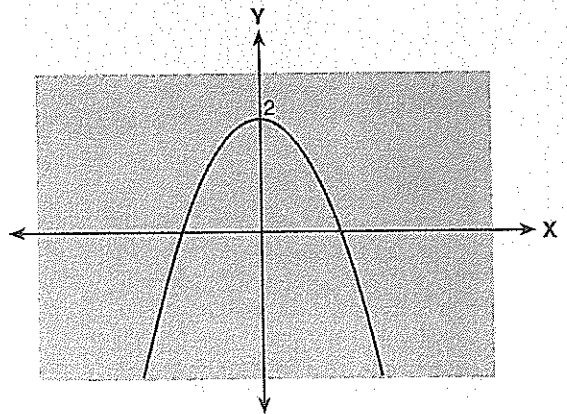
**D**



**E**



**F**



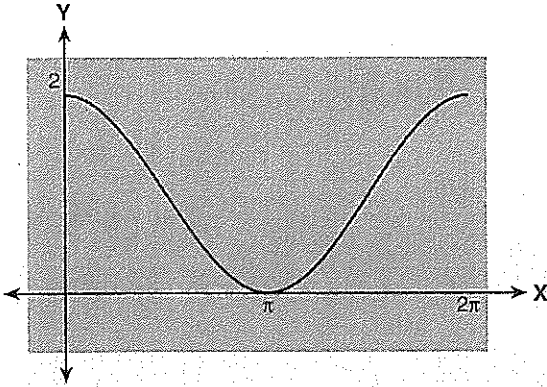
- 1 As  $x \rightarrow \infty$ ,  $y \rightarrow 2$
- 2 The equation of the curve is  $y = \frac{1}{x} - 2$
- 3 The graph shown is  $y = x^2 - 4x + 4$
- 4 This function is increasing when  $x < 0$
- 5 The x-intercept is  $(2, 0)$
- 6 The function is  $y = x^2 - 2$
- 7 The vertical asymptote is  $x = 2$
- 8 The range is  $y \geq -2$ ,  $y \in R$
- 9 The function is  $y = \frac{1}{x} + 2$
- 10 The function is always decreasing
- 11 The curve cuts the x-axis at  $(0.5, 0)$
- 12 The function is a hyperbola
- 13 The curve is parabolic
- 14 The y-intercept is  $(0, -2)$
- 15 The function is increasing when  $x > 0$
- 16 The function is  $y = \frac{2x + 1}{x}$
- 17 The curve has no y-intercept
- 18 The equation of the curve is  $y = 2 - x^2$
- 19 The function is  $y = \frac{4}{x - 2}$
- 20 The graph shows  $y = (x - 2)^2$
- 21 The domain is  $x \in R$
- 22 The horizontal asymptote is  $y = 2$
- 23 There are two x-intercepts. One is  $(\sqrt{2}, 0)$
- 24 As  $x \rightarrow \infty$ ,  $y \rightarrow -2$



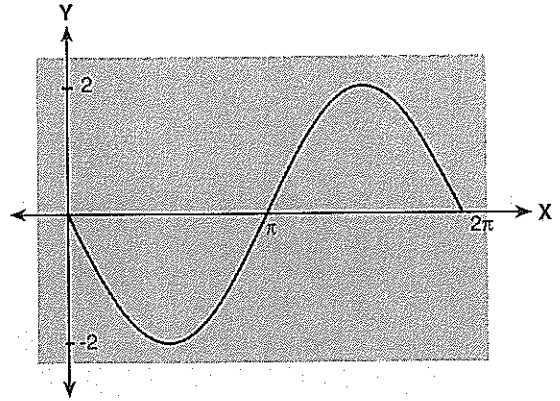
# Trigonometric curves

Match four appropriate statements from page eight with each graph.

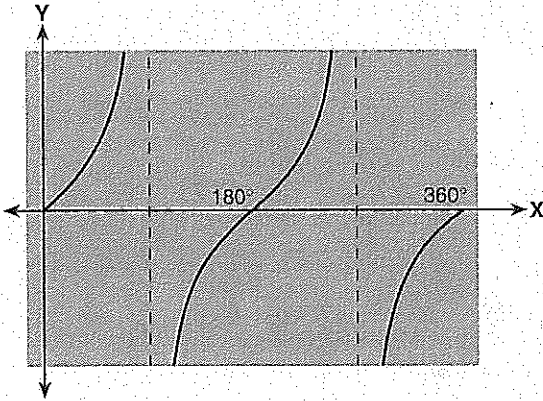
**A**



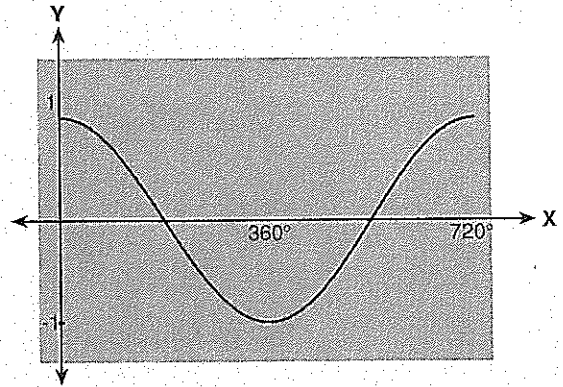
**B**



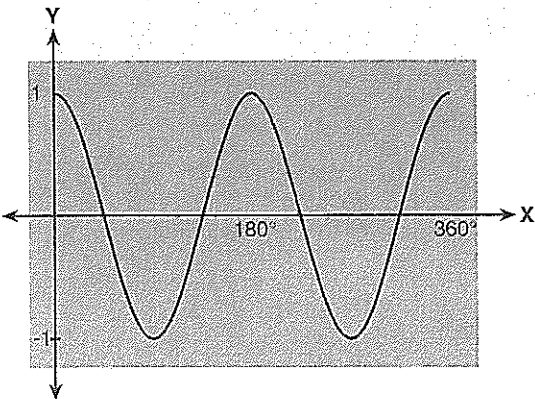
**C**



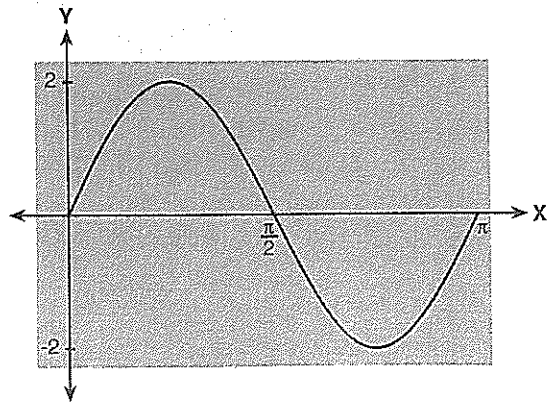
**D**



**E**



**F**



1

There is a turning point at  $x = 90^\circ$

2

The function is  $y = 2 \sin 2x$

3

The function is  $y = \cos \frac{x}{2}$

4

This function is  $y = -2 \sin x$

5

The function is periodic

6

The minimum point is  $(\pi, 0)$

7

The curve is always increasing

8

The period of the function is  $720^\circ$

9

The function is  $y = 1 + \cos x$

10

The function is  $y = \tan x$

11

The function has a discontinuity at  $x = 90^\circ$

12

The function is decreasing when  $180^\circ < x < 270^\circ$

13

The amplitude is 1

14

The amplitude of the curve is 2

15

The maximum point is  $(\frac{3\pi}{2}, 2)$

16

The y-intercept is  $(0, 1)$

17

There are four x-intercepts

18

This function has an axis of symmetry where  $x = \pi$

19

The period is  $180^\circ$

20

The curve is  $y = \cos 2x$

21

The range of the function is  $-2 \leq y \leq 2, y \in R$

22

The domain is  $0 \leq \theta \leq \pi$

23

The curve has a negative gradient for  $0 < x < \pi$

24

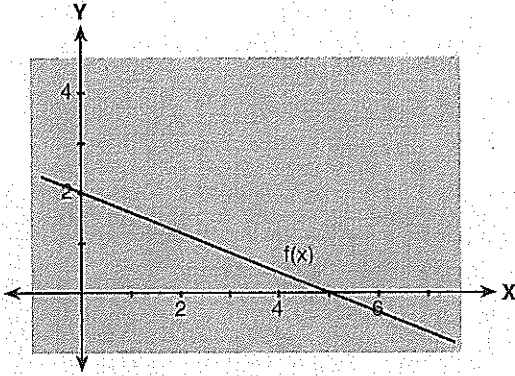
The period is  $2\pi$



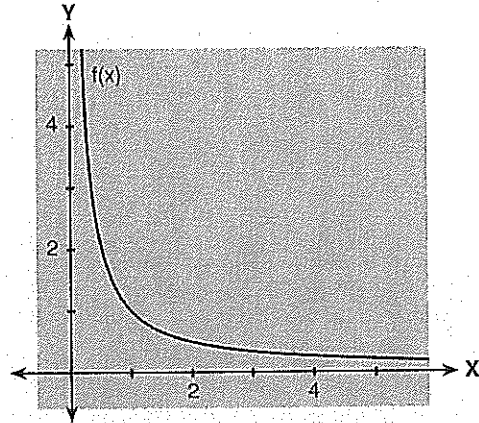
## Gradients and graphs

Match four appropriate statements from page ten with each graph.

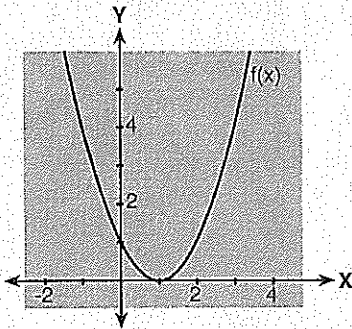
**A**



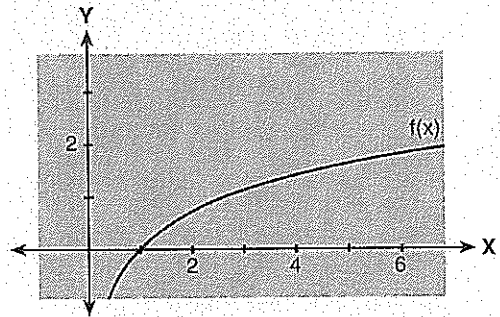
**B**



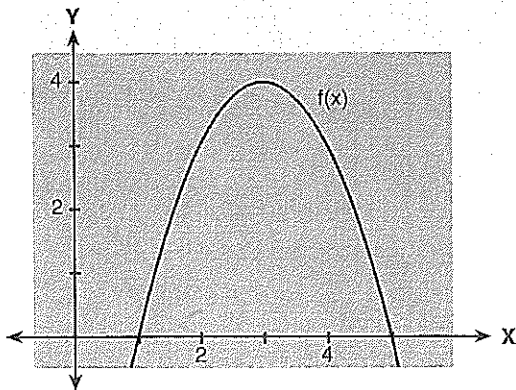
**C**



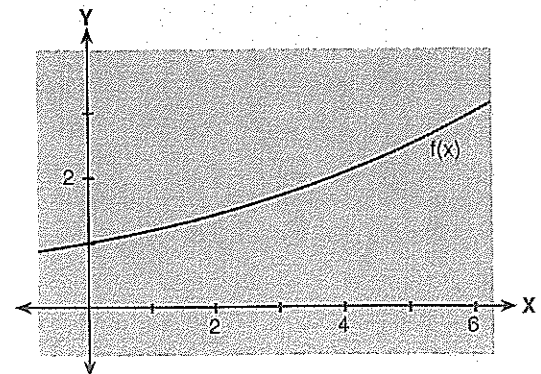
**D**



**E**



**F**





1 As  $x$  increases  $f(x)$  increases

2  $f(x) < 0$  when  $x < 1$  and  $x > 5$

3 There is a minimum at  $(1, 0)$

4  $f(5) = 0$

5  $f(x) = \ln x$

6  $\frac{dy}{dx} > 0$  when  $x > 1$

7 The gradient decreases as  $x$  gets larger

8 The gradient of the graph is negative

9  $\frac{dy}{dx} < 0$  and  $\frac{d^2y}{dx^2} > 0$

10 The gradient is constant

11 The curve is symmetrical about  $x = 3$

12 The slope is negative when  $x < 1$

13 The equation of the function is  $y = \frac{-2}{5}x + 2$

14  $\frac{d^2y}{dx^2} < 0$  for all values of  $x$

15  $\frac{dy}{dx} = 0$  at  $x = 3$

16  $f(x)$  decreases as  $x$  increases

17 This is an exponential curve

18 The limit of the function as  $x \rightarrow \infty$  is 0

19 The gradient of  $f(x)$  is positive

20 As  $x$  increases,  $f(x)$  increases

21  $\frac{dy}{dx} > 0$  for all  $x$

22  $f(0) = 1$

23 The gradient increases as  $x$  increases

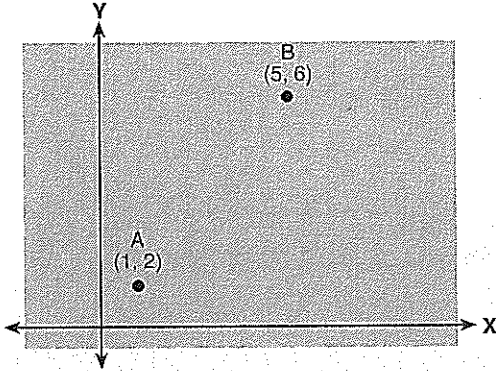
24  $y' = 2x - 2$



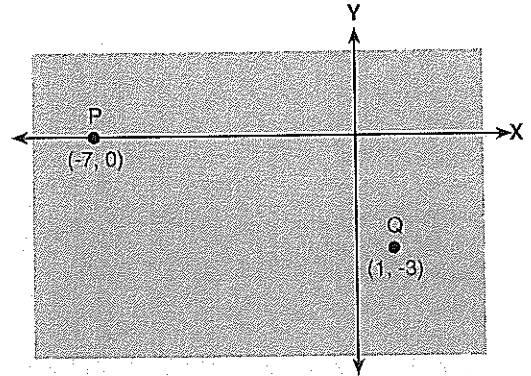
# Coordinate geometry (1)

Match four appropriate statements from page twelve with each set of points.

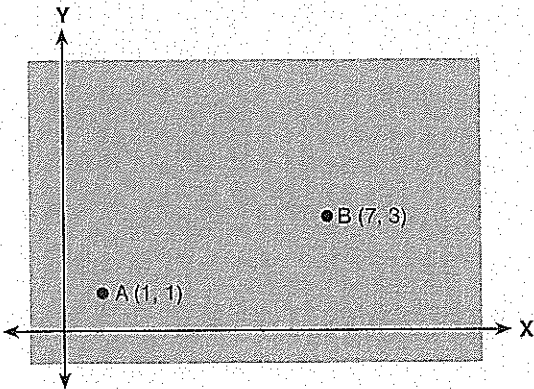
**A**



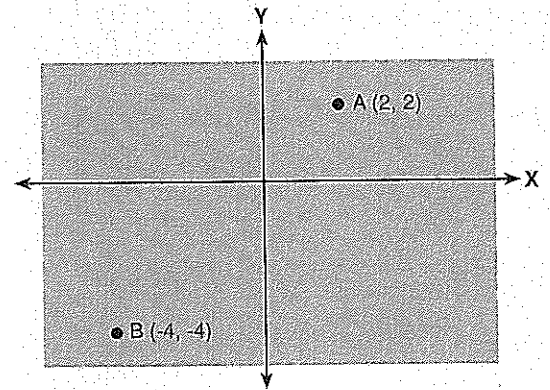
**B**



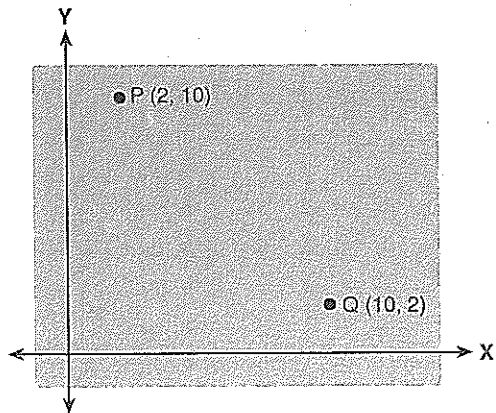
**C**



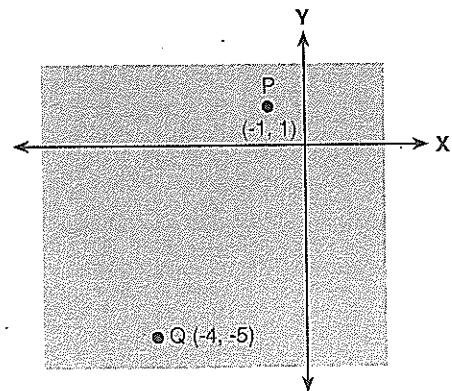
**D**



**E**



**F**



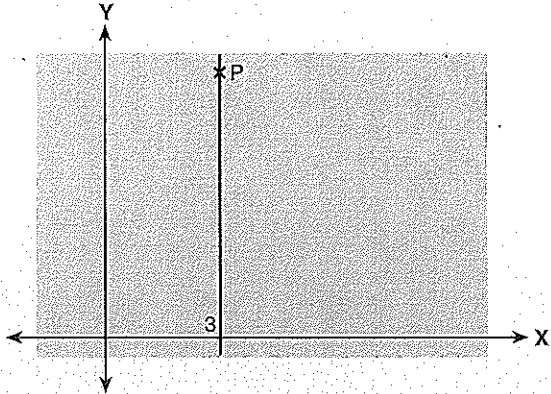
- 1  $\overline{PQ}$  is parallel to the line  $y = 2 - x$
- 2  $A$  and  $B$  are in the same quadrant
- 3  $\overline{AB}$  goes through the origin
- 4  $\overline{PQ}$  has a negative gradient
- 5 The midpoint of  $\overline{AB}$  is  $(4, 2)$
- 6 The length of  $\overline{AB}$  is  $\sqrt{32}$  units
- 7  $B$  is in the third quadrant
- 8  $\overline{PQ}$  has a positive gradient
- 9  $P$  is on the  $x$ -axis
- 10 The gradient of  $\overline{AB}$  is 1
- 11  $(6, 6)$  is halfway between  $P$  and  $Q$
- 12  $\overline{PQ}$  is  $\sqrt{73}$  units long
- 13 The midpoint of  $Q$  and  $P$  is  $(-3, -1.5)$
- 14 The length  $\overline{AB}$  is  $\sqrt{40}$  units
- 15 The distance from  $P$  to  $Q$  is  $\sqrt{45}$  units
- 16  $P$  and  $Q$  are not in the same quadrant
- 17  $\overline{AB}$  cuts the  $y$ -axis at  $(0, 1)$
- 18 The midpoint of  $\overline{AB}$  is  $(3, 4)$
- 19  $\overline{PQ}$  has length  $\sqrt{(10 - 2)^2 + (2 - 10)^2}$
- 20 The length of  $\overline{AB}$  is  $\sqrt{72}$  units
- 21  $\overline{PQ}$  is perpendicular to the line  $y = -0.5x + 3$
- 22 The slope of  $\overline{AB}$  is  $\frac{1}{3}$
- 23  $A$  is equidistant from the  $x$  and  $y$  axes
- 24  $P$  is closer to the  $y$ -axis than  $Q$



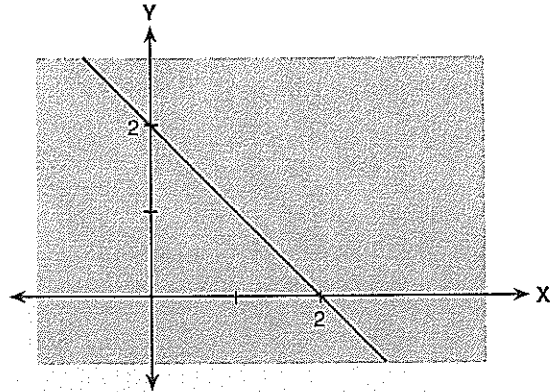
# Coordinate geometry (2)

Match four appropriate statements from page fourteen with each graph.

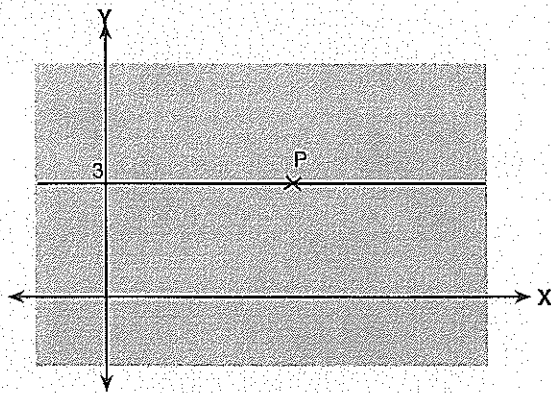
**A**



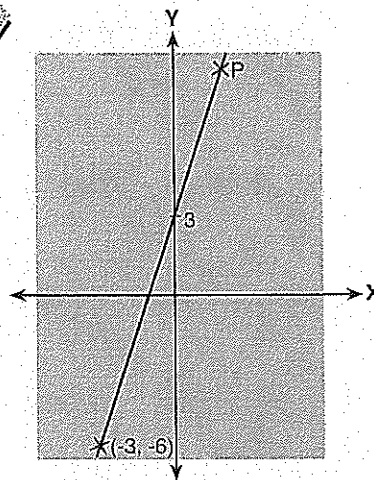
**B**



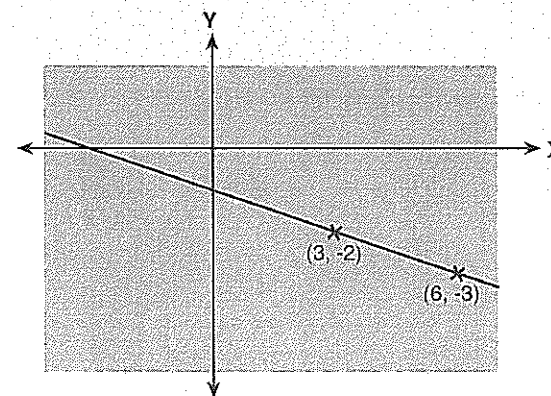
**C**



**D**



**E**



**F**

